Electroweak Corrections at the LHC

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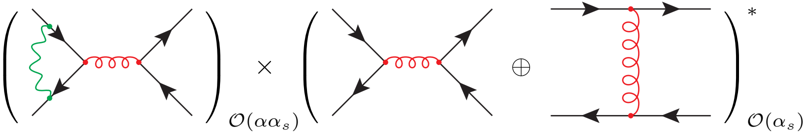
Outline

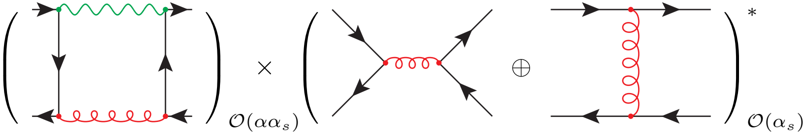
- Introduction
  - Motivation
  - Sudakov logarithmic corrections
- Application to processes
  - Neutral current Drell-Yan process
  - Top pair production
  - Dijets production
- Conclusion and outlook

Example of electroweak corrections

- Electroweak corrections to dijet production ($\mathcal{O}(\alpha\alpha_s^2)$)
  - EW vertex correction



  - EW box correction



Electroweak corrections enhanced via Sudakov logarithms

- Electroweak corrections at the LHC can be enhanced at high energies due to soft/collinear radiation of W and Z bosons.
- When all kinematic invariants $r_{ij} = (p_j + p_k)^2$ are much larger than the heavy particles in the loop, i.e., $|r_{ij}| \sim Q^2 \gg M_W^2 \sim M_Z^2 \sim M_H^2 \sim m_t^2$, electroweak corrections are dominated by Sudakov-like corrections:

$$\boxed{\alpha_W^l \log^n(Q^2/M_W^2)} \quad (n \leq 2l-1, \ \alpha_W = \frac{\alpha}{4\pi s_W^2})$$

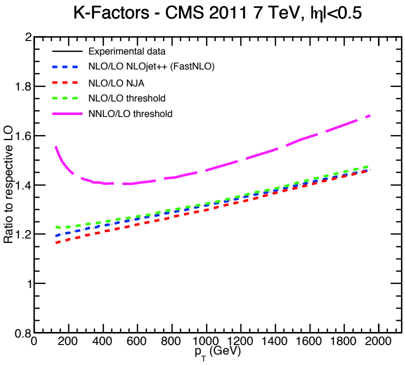
  - $Q = 1\text{TeV}$,
    $\alpha_W \log^2(Q^2/M_W^2) \sim \boxed{6.6\%}$, $\alpha_W \log(Q^2/M_W^2) \sim \boxed{1.3\%}$
  - $Q = 14\text{TeV}$,
    DL $\sim \boxed{27\%}$, SL $\sim \boxed{2.6\%}$

Why electroweak corrections?

- The inclusion of EW corrections in LHC predictions is important for the search of new physics in tails of distributions, e.g., search for W', Z', non-standard couplings
- It is also important for contraints on PDFs measurement

EW NLO $\boxed{\mathcal{O}(\alpha)}$ is expected comparable with QCD NNLO $\boxed{\mathcal{O}(\alpha_s^2)}$

S. Carrazza, J. Pires [arXiv:1407.7031]
QCD k-factor for LHC jet prediction



Why electroweak corrections?

- Calculations of electroweak corrections are often not readily available in public codes and can quickly become complicated (and CPU intensive) for high multiplicities.
- As a first step to improve predictions for the LHC at high energies, one could implement the Sudakov approximation of electroweak corrections.
  Example: Weak Sudakov corrections to $Z + \leq 3$ jets in Alpgen
  M. Chiesa *et al*, PRL111 (2013).
  See also a recent proposal to add EW corrections to HERWIG:
  [http://arxiv.org/pdf/1401.3964.pdf] Link Here
- Our goal is to implement EW corrections in MCFM so that they become readily available to the experimental community and can be studied together with the already implemented QCD corrections.

Public codes of the EW corrections to DY-like process

- Complete EW $\mathcal{O}(\alpha)$ corrections: HORACE, RADY, SANC, W/ZGRAD2 U. Baur *et al*, PRD65 (2002); C. M. Carloni Calame *et al*, JHEP05 (2005); U. Baur, D. Wackeroth, PRD70 (2004); S. Dittmaier, M. Krämer, PRD65 (2002); A. Andonov *et al*, EPJC46 (2006); Arbuzov *et al*, EPJC54 (2008); S. Dittmaier, M. Huber, JHEP60 (2010).
- Multiple final-state photon radiation: HORACE, RADY, WINHAC, PHOTOS W. Placzek *et al*, EPJC29 (2003); C. M. Carloni Calame *et al*, PRD69 (2004); S. Brensing *et al*, PRD77 (2008).
- NLO EW corrections to W production in POWHEG C. Bernaciak, D. Wackeroth, PRD85 (2012).
- NLO EW corrections to Z production in POWHEG L. Barze *et al*, EPJC73 (2013).
- NLO EW corrections to Z production in FEWZ with NNLO QCD Ye Li, F. Petriello, PRD86 (2012).

Implementation in MCFM

- We will provide both the Sudakov approximation for EW corrections valid at high energies and the complete 1-loop weak corrections to be able to quantify the goodness of the approximation.
  - NC Drell Yan process
    - I Weak Sudakov correction ✓
    - II Exact NLO weak correction ✓
  - Top-pair production (*New in public tools*)
    - I Weak Sudakov correction ✓
    - II Exact NLO weak correction ✓
  - Dijet production (*New in public tools*)
    - I Weak Sudakov correction – ongoing
    - II Exact NLO weak correction – ongoing
- For a recent review of status of EW corrections see: Link Here [https://phystev.in2p3.fr/wiki/_media/2013:groups:lh13_ew.pdf]

Sudakov logarithms calculations

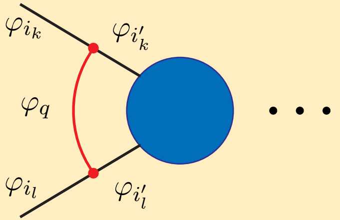
- Vertex Part at Very High Energies in QED
  V. V. Sudakov, Soviet Phys. JETP3 (1956) 65
- Some Refs. for the general Sudakov logarithmic corrections
  P. Ciafaloni, D. Comelli, PLB446 (1999), arXiV:hep-ph/9809321; M. Beccaria *et al*, PRD61 (2000), arXiv:hep-ph/9906319; J. H. Kühn, A. A. Penin, arXiv:hep-ph/9906545; M. Melles, Phys. Rept.375(2003), arXiv:hep-ph/0104232; A. Denner, S. Pozzorini, EPJC18 (2001), arXiv:hep-ph/0010201; A. Denner, S. Pozzorini, EPJC21(2001), arXiv:hep-ph/0104127; S. Pozzorini, arXiv:hep-ph/0201077; W. Beenakker, A. Werthenbach, NPB630 (2002), arXiv:hep-ph/0112030; A. Denner *et al*, JHEP0811 (2008), arXiv:0809.0800.

▶ The general algorithm of Denner and Pozzorini is adopted in the implementation in MCFM

Double Logarithmic (DL) Mass Singularities

Eikonal approximation $\mathrm{q}^\mu \to \mathrm{x} \cdot \mathrm{p}^\mu_{\mathrm{k,l}}$, and $x \to 0$

$$\delta^{DL}\mathcal{M}^{\varphi_{i_1}\cdots\,\varphi_{i_n}} = \sum_{\varphi_q} \sum_{k,l=1,(l<k)}^{n} \Big[\qquad \Big]_{\text{eik.}}$$



$$= \sum_{\varphi_q} \sum_{k=1}^{n} \sum_{l<k} \int \frac{d^4q}{(2\pi)^4} \frac{-4ie^2 p_k p_l I^{\varphi_q}_{i'_k i_k}(k) I^{\bar{\varphi}_q}_{i'_l i_l}(l) \mathcal{M}_0^{i_1\ldots i'_k \ldots i'_l \ldots i_n}}{[q^2 - M^2_{\varphi_q}][(p_k+q)^2 - m^2_{\varphi_{i'_k}}][(p_l-q)^2 - m^2_{\varphi_{i'_l}}]}$$

$\varphi_q = V^a = A, Z, W^\pm$, others are mass supressed (such as $\phi^\pm$, χ^0, H).
$ieI^{\varphi_q}_{i'_k i_k}$ is the coupling of the vertex $\varphi_q \bar{\varphi}_{i'_k} \varphi_{i_k}$.

Double Logarithmic (DL) Mass Singularities

3-point scalar integral

$$\mathbf{C_0} = \int \frac{d^4q}{(2\pi)^4} \frac{1}{[q^2 - M^2_{V^a}][(p_k+q)^2 - m^2_{\varphi_{i'_k}}][(p_l-q)^2 - m^2_{\varphi_{i'_l}}]},$$

$$\mathbf{C_0} \xrightarrow{r_{kl} \gg p^2_{k,l}} \frac{1}{r_{kl}} \left\{ \frac{1}{2} \log^2 \left(\frac{-r^2_{kl} - i\varepsilon}{M^2_{V^a} - i\varepsilon} \right) + \sum_{m=k,l} \mathbf{I}_c(p^2_m, M^2_{V^a}, M^2_{i'_m}) \right\}$$

where $r_{kl} = (p_k + p_l)^2 \approx 2p_k \cdot p_l$,

$$\mathbf{I}_c = \sum_{\pm} \mathrm{Li}_2 \left(\frac{2p^2_m}{M^2_{V^a} - m^2_{i'_m} + p^2_m \pm \kappa(p^2_m, M^2_{V^a} - i\varepsilon, m^2_{i'_m} - i\varepsilon)} \right),$$

$$\kappa(a,b,c) = \sqrt{a^2 + b^2 + c^2 - 2ab - 2ac - 2bc}.$$

Double Logarithmic (DL) Mass Singularities

Discarding imaginary and finite pieces that do not increase with energy, we obtain a symmetric DL proportional to the Born amplitude:

$$\delta^{DL}_{V^a}\mathcal{M}^{i_1\ldots i_n} = \frac{\alpha}{4\pi}\left[\log^2\frac{|r_{kl}|}{M^2_{V^a}} + 2\sum_{m=k,l}\mathbf{I}_c(p_m^2, M^2_{V^a}, m^2_{i'_m})\right] I^{V^a}_{i'_k i_k} I^{\bar{V}^a}_{i'_l i_l}\mathcal{M}_0^{i_1\ldots i'_k\ldots i'_l\ldots i_n}$$

$\mathbf{I}_c(p_m^2, M^2_{V^a}, m^2_{i'_m})$ is significant only if $V^a = A$.

Universal DL

$$\delta^{DL}\mathcal{M}^{i_1\ldots i_n} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\sum_{V^a} C^{\mathrm{ew}}_{i'_k i_k}\mathcal{M}_0^{i_1\ldots i'_k\ldots i_n}$$

$$\times\left[\frac{1}{2}\log^2\frac{|r_{kl}|}{M^2_{V^a}} - \frac{1}{2}\delta_{V^aA}\log^2\frac{M^2_{i_k}}{\lambda^2} + 2(1-\delta_{V^aA})\mathbf{I}_c(p_m^2, M^2_{V^a}, m^2_{i'_m})\right]$$

Note: we have written $\sum_{V^a}\sum_{i'_k,i'_l} I^{V^a}_{i'_k i_k} I^{\bar{V}^a}_{i'_l i_l}\mathcal{M}_0^{i_1\ldots i'_k\ldots i'_l\ldots i_n} = \sum_k C^{\mathrm{ew}}_{i'_k i_k}\mathcal{M}_0^{i_1\ldots i'_k\ldots i_n}$, where $C^{\mathrm{ew}}_{i'_k i_k} = \sum_{V^a} I^{V^a} I^{\bar{V}^a}$ is the Casimir operator.

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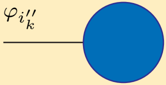
Collinear Mass Singularities

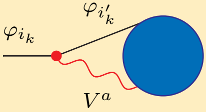
Virtual gauge boson goes collinear when $\mathrm{q}^\mu \to \mathrm{x} \cdot \mathrm{p}_\mathrm{k}^\mu$

$$\delta^{coll} \mathcal{M}^{\varphi_{i_k}}(p_k) = \delta^{coll}_{\varphi_{i''_k}\varphi_{i_k}} \cdot \mathcal{M}^{\varphi_{i''_k}} = \sum_{\varphi_{i''_k}}$$

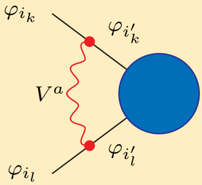


$$\delta^{coll}_{\varphi_{i''_k}\varphi_{i_k}}$$

$$= \sum_{V^a} \sum_{\varphi_{i'_k}} \Bigg\{ \Bigg[$$



$$\Bigg]_{\text{trunc.}} - \sum_{l \neq k} \sum_{\varphi_{i'_l}} \Bigg[$$



$$\Bigg]_{\text{eik.}} \Bigg\}_{\text{coll.}}$$

Subtracting the soft and collinear eikonal contributions which have been accounted for in DL.

Collinear Mass Singularities

$$\delta^{coll}_{\varphi_{i''_k}\varphi_{i_k}} \cdot \mathcal{M}^{\varphi_{i''_k}} = \sum_{V^a}\sum_{\varphi_{i'}} \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{-iK_{\varphi_i} e^2 I^{V^a}_{\varphi_{i''}\varphi_{i'}} I^{\bar{V}^a}_{\varphi_{i'}\varphi_i}}{[q^2 - M^2_{V^a}][(p-q)^2 - M^2_{\varphi_{i'}}]},$$

$$K_{\varphi_i} = \begin{cases} 1, & \varphi_i \text{ is a scalar or transverse gauge boson,} \\ 2, & \varphi_i \text{ is a fermion.} \end{cases}$$

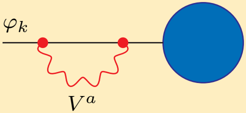
Collinear single logarithmic corrections

$$\delta^{coll}_{\varphi_{i''_k}\varphi_{i_k}} \cdot \mathcal{M}^{\varphi_{i''_k}} \overset{\mathrm{LA}}{=} \frac{\alpha}{4\pi} K_{\varphi_i} \Bigg\{ C^{\mathrm{ew}}_{\varphi_{i''}\varphi_i} \log\frac{\mu^2}{M_W^2} + \delta_{\varphi_{i''}\varphi_i} Q^2_{\varphi_i} \log\frac{M_W^2}{M^2_{\varphi_i}}$$
$$- \sum_{V^a}\sum_{\varphi_{i'}} I^{V^a}_{\varphi_{i''}\varphi_{i'}} I^{\bar{V}^a}_{\varphi_{i'}\varphi_i} \log\left(\frac{\max(M_W^2, M^2_{\varphi_i}, M^2_{\varphi_{i'}})}{M_W^2}\right) \Bigg\} \cdot \mathcal{M}^{\varphi_{i''_k}}.$$

Logarithmic corrections from renormalization

Wave Function Renormalization

$$\delta^{\mathrm{WF}}\mathcal{M}^{\varphi_1\cdots\varphi_n}(p_1,\cdots,p_n) = $$



$$= \sum_{k=1}^{n}\sum_{\varphi'_k}\mathcal{M}_0^{\varphi_1\cdots\varphi'_k\cdots\varphi_n}(p_1,\cdots,p_n)\cdot\delta^{\mathrm{WF}}_{\varphi'_k\varphi_k},$$

$$\delta^{\mathrm{WF}}_{\varphi'_k\varphi_k} = \frac{1}{2}\delta \mathrm{Z}_{\varphi'_k\varphi_k}, \quad \text{For all but } V^a_L,$$

$$\delta^{\mathrm{WF}}_{V^{a'}_L V^a_L} = \frac{1}{2}\delta \mathrm{Z}_{\Phi^{a'}\Phi^a} + \delta \mathrm{A}^{V^a}\cdot\delta_{V^{a'}V^a}$$

Logarithmic corrections from renormalization

Parameter Renormalization

$$\delta^{PR}\mathcal{M}^{\varphi_1\cdots\varphi_n} = \sum_{\lambda_i} \frac{\partial \mathcal{M}_0^{\varphi_1\cdots\varphi_n}}{\partial \lambda_i}\delta\lambda_i\Big|_{\mu^2=\hat{s}}, \quad \lambda_{0,i} = \hat{\lambda}_i + \delta\lambda_i$$

Parameters: $\boxed{e}$, $\boxed{s_W,\, c_W}$, $\boxed{h_t = \frac{m_t}{M_W}}$, $\boxed{h_H = \frac{M_H^2}{M_W^2}}$.

Setting $\mu^2 = \hat{s}$

$$\log\frac{\hat{s}}{\mu^2} + \log\frac{\mu^2}{\mu_R^2} = \log\frac{\hat{s}}{\mu_R^2}, \mu_R = \mu_F = \frac{M_W}{2}, M_W, 2M_W$$

free to choose of $\mu^2(\mu_F^2) = \hat{s}$

loop *CT* *combination*

Leading approximation(LA) in renormalization CTs

- FRCs to External legs
  1. Chiral fermions

$$\delta Z_{f^\kappa_{j,\sigma} f^\kappa_{j,\sigma}} \stackrel{\mathrm{LA}}{=} \frac{\alpha}{4\pi}\left[-C^\kappa_{f_{j,\sigma}} \log\frac{\mu^2}{M_W^2} + Q_{f^2_{j,\sigma}}\left(2\log\frac{M_W^2}{\lambda^2} - 3\log\frac{M_W^2}{m^2_{f_{j,\sigma}}}\right)\right] + \delta Z^{top}_{f^\kappa_{j,\sigma}},$$

$$\delta Z^{top}_{f^\kappa_{j,\sigma}} \stackrel{\mathrm{LA}}{=} \frac{\alpha}{4\pi}\left[\frac{1}{4s_W^2}\left((1+\delta_{\kappa R})\frac{m^2_{f_{j,\sigma}}}{M_W^2} + \delta_{\kappa L}\frac{m^2_{f_{j,-\sigma}}}{M_W^2}\right)\right]$$

- Parameter renormalization
  1. Mixing-angle renormalization

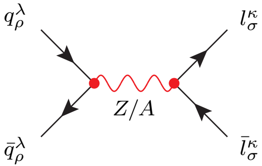
$$\frac{\delta c_W^2}{c_W} = \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \stackrel{\mathrm{LA}}{=} \frac{s_W}{c_W} b^{\mathrm{ew}}_{AZ} l(\mu^2)$$

  2. Charge renormalization

$$\begin{aligned} \delta Z_e &= -\frac{1}{2}\left[\delta Z_{AA} + \frac{s_W}{c_W}\delta Z_{AZ}\right] \\ &\stackrel{\mathrm{LA}}{=} -\frac{1}{2} b^{\mathrm{ew}}_{AA} l(\mu^2) + \frac{2}{3}\sum_{f_{\sigma,i}\neq t} N^f_{\mathrm{C}} Q_{f_{\sigma,i}} l(M_W^2, m_{f^2_{\sigma,i}}) \end{aligned}$$

Sudakov approximation to Drell-Yan process

Process under consideration: $\bar{q}_\rho^\lambda q_\rho^\lambda l_\sigma^\kappa \bar{l}_\sigma^\kappa \to 0$



Born amplitude

$$\mathcal{M}^{\bar{q}_\rho^\lambda q_\rho^\lambda l_\sigma^\kappa \bar{l}_\sigma^\kappa} = e^2 R_{q_\rho^\lambda l_\sigma^\kappa} \frac{\mathcal{A}}{\hat{s}} + \mathcal{O}\left(\frac{M_Z^2}{\hat{s}}\right),$$

$$R_{\phi_i \phi_k} := \sum_{N=Z,A} I_{\phi_i}^N I_{\phi_k}^N = \frac{1}{4c_W^2} Y_{\phi_i} Y_{\phi_k} + \frac{1}{s_W^2} T_{\phi_i}^3 T_{\phi_k}^3,$$

$Y_{\phi_{i,k}}$ — weak hypercharge; $T_{\phi_{i,k}}^3$ — 3rd component of weak isospin.

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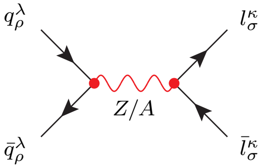


Born amplitude

$$\mathcal{M}^{\bar{q}^{\lambda}_{\rho} q^{\lambda}_{\rho} l^{\kappa}_{\sigma} \bar{l}^{\kappa}_{\sigma}} = e^2 R_{q^{\lambda}_{\rho} l^{\kappa}_{\sigma}} \frac{\mathcal{A}}{\hat{s}} + \mathcal{O}(\frac{M_Z^2}{\hat{s}}),$$

$$R_{\phi_i \phi_k} := \sum_{N=Z,A} I^N_{\phi_i} I^N_{\phi_k} = \frac{1}{4c_W^2} Y_{\phi_i} Y_{\phi_k} + \frac{1}{s_W^2} T^3_{\phi_i} T^3_{\phi_k},$$

$Y_{\phi_{i,k}}$ — weak hypercharge; $T^3_{\phi_{i,k}}$ — 3rd component of weak isospin.

Sudakov approximation to Drell-Yan process

Leading and subleading soft-collinear corrections

$$\delta^{LSC}_{\bar{q}^\lambda_\rho q^\lambda_\rho l^\kappa_\sigma \bar{l}^\kappa_\sigma} = -\sum_{f^\mu_\tau = q^\lambda_\rho, l^\kappa_\sigma} \left[C^{\mathrm{ew}}_{f^\mu_\tau} L(\hat{s}) - 2(I^Z_{f^\mu_\tau})^2 \log \frac{M_Z^2}{M_W^2} l_Z + Q^2_{f_\tau} L^{em} \cancel{(\hat{s}, \lambda^2, m^2_{f_\tau})} \right],$$

$$\delta^{SSC}_{\bar{q}^\lambda_\rho q^\lambda_\rho l^\kappa_\sigma \bar{l}^\kappa_\sigma} = -l(s) \left[4R_{q^\lambda_\rho l^\kappa_\sigma} \log \frac{\hat{t}}{\hat{u}} + \frac{\delta_{\lambda L}\delta_{\kappa L}}{s_w^4 R_{q^\lambda_\rho l^\kappa_\sigma}} \left(\delta_{\rho\sigma} \log \frac{|\hat{t}|}{s} - \delta_{-\rho\sigma} \log \frac{|\hat{u}|}{s} \right) \right]$$

$$-4Q_{q_\rho} Q_{l_\sigma} \cancel{l(M_W^2, \lambda^2)} \log \frac{\hat{t}}{\hat{u}}$$

$$L(\hat{s}) := \frac{\alpha}{4\pi} \log^2 \frac{\hat{s}}{M_W^2}, \quad l_Z = l(\hat{s}) := \frac{\alpha}{4\pi} \log \frac{\hat{s}}{M_W^2}.$$

Sudakov approximation to Drell-Yan process

Collinear or soft SL corrections

$$\delta^{C}_{\bar{q}^{\lambda}_{\rho} q^{\lambda}_{\rho} l^{\kappa}_{\sigma} \bar{l}^{\kappa}_{\sigma}} = \sum_{f^{\mu}_{\tau} = q^{\lambda}_{\rho}, l^{\kappa}_{\sigma}} \left[3C^{\mathrm{ew}}_{f\mu} l_C - \frac{1}{4s^2_W} \left((1+\delta_{\mu R}) \frac{m^2_{f_\tau}}{M^2_W} + \delta_{\mu L} \frac{m^2_{f_{-\tau}}}{M^2_W} \right) l_{Yuk} + 2Q^2_{f_\tau} \cancel{l^{em}}(m^2_{f_\tau}) \right]$$

Parameter renormalization corrections

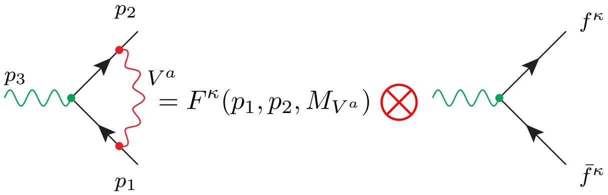
$$\delta^{PR}_{\bar{q}^{\lambda}_{\rho} q^{\lambda}_{\rho} l^{\kappa}_{\sigma} \bar{l}^{\kappa}_{\sigma}} = \left[\frac{s_W}{c_W} b^{\mathrm{ew}}_{AZ} \Delta_{q^{\lambda}_{\rho} l^{\kappa}_{\sigma}} - b^{\mathrm{ew}}_{AA} \right] l_{PR} + \cancel{2\delta Z^{em}_{e}}$$

$$\Delta_{\phi_i \phi_k} := \frac{-\frac{1}{4c^2_W} Y_{\phi_i} Y_{\phi_k} + \frac{c^2_W}{s^4_W} T^3_{\phi_i} T^3_{\phi_k}}{R_{\phi_i \phi_k}}$$

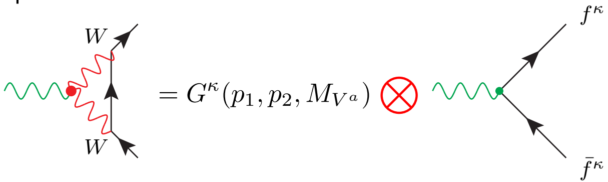
$$l_C = l_{Yuk} = l_{PR} = l(\hat{s}) := \frac{\alpha}{4\pi} \log \frac{\hat{s}}{M^2_W}, \quad b^{\mathrm{ew}}_{AZ} = -\frac{19+22s^2_{\mathrm{W}}}{6s^2_{\mathrm{W}} c^2_{\mathrm{W}}}, \quad b^{\mathrm{ew}}_{AA} = -\frac{11}{3}.$$

One-loop weak correction to Drell-Yan

- Vertex corrections
  - virtual gauge boson exchange between fermion external legs



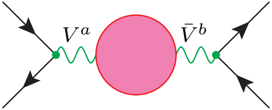
  - internal W-pair in vertex



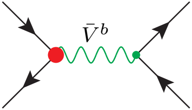
  - scalar boson correction are suppressed by $m_{f^{\kappa}}$

One-loop weak correction to Drell-Yan

- Self energy corrections



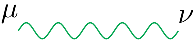
$$= \frac{\hat{\Sigma}_T^{V^a\bar{V}^b}(\hat{s})}{\hat{s} - M_{V^a}^2 + iM_{V^a}\Gamma_{V^a}} \otimes$$



 denotes the coupling: $\mathrm{I}_{f_\tau^\sigma}^{V^a}$ v.s. the final state fermion coupling is $\mathrm{I}_{f_\tau^\sigma}^{\bar{V}^b}$.

- Z decay width in vertex and self-energy corrections

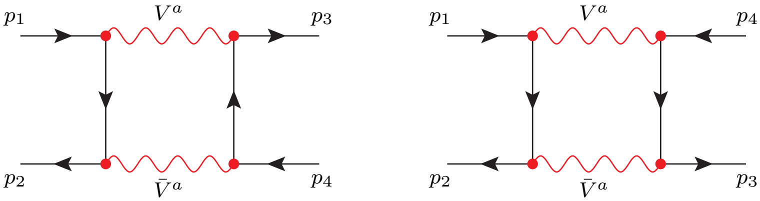
Constant Z width Γ_Z, no Z or W width in bubble or triangle integrals:



$$= \frac{-ig_{\mu\nu}}{\hat{s} - M_Z^2 + iM_Z\Gamma_Z}$$

One-loop weak correction to Drell-Yan

- Box corrections



$$V^a = Z,\ W^\pm$$

- Z width in box correction

Constant Z width in Born, no Z or W width in box

$$d\hat{\sigma}_{\text{box}} = 2\text{Re}\left(\mathcal{M}_{\text{box}} \times \mathcal{M}^*_{\text{Born}}\right) \propto \frac{\hat{s} - M_Z^2}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Input parameter schemes for α

- $\boxed{\alpha(0)\text{-scheme:}}$ use $\alpha(0)$ everywhere; the relative corrections sensitively depend on the light-fermion masses via $\alpha \log m_f$ terms that enter the charge renormalization.
- $\boxed{\alpha(M_Z)\text{-scheme:}}$ the relative corrections have contributions from $\Delta\alpha(M_Z)$, which accounts for the running of the electromagnetic coupling from $Q = 0$ to $Q = M_Z$ and cancels $\alpha \log m_f$ terms; free of light-fermion mass dependence.
- $\boxed{G_\mu\text{-scheme:}}$ use the Fermi constant G_μ; corresponding electromagnetic coupling $\alpha_\mu = \sqrt{2} G_\mu M_W^2 (1 - M_W^2/M_Z^2)/\pi$; relative corrections have contributions from Δr, which describes the radiative corrections to muon decay. And it is also free of light-fermion mass dependence.

  [Dittmaier and Huber, JHEP 1001 (2010) 060; arXiv: 0911.2329]

The input parameter setup

- Both calculations are included in MCFM
  - Exact
  - Sudakov

The input parameter setup in MCFM:

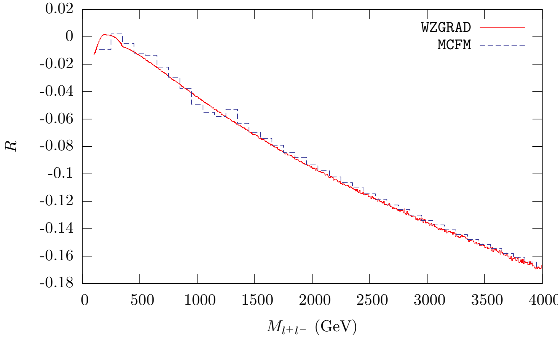
$$G_\mu = 1.16639 \times 10^{-5}\,\mathrm{GeV}^{-2},\ \sin^2\theta_W = 1 - M_W^2/M_Z^2,$$
$$\alpha_\mu = 1/132.5605045,\ \Gamma_Z = 2.4952\,\mathrm{GeV},\ \cos^2\theta_W = M_W^2/M_Z^2,$$
$$M_Z = 91.1876\,\mathrm{GeV},\ M_W = 80.425\,\mathrm{GeV},\ M_H = 120\,\mathrm{GeV},$$
$$m_e = 0.51099892\,\mathrm{MeV},\ m_\mu = 105.658369\,\mathrm{MeV},\ m_\tau = 1.777\,\mathrm{GeV},$$
$$m_u = 66\,\mathrm{MeV},\ m_c = 1.2\,\mathrm{GeV},\ m_t = 173.2\,\mathrm{GeV},$$
$$m_d = 66\,\mathrm{MeV},\ m_s = 150\,\mathrm{MeV},\ m_b = 4.6\,\mathrm{GeV},$$
$$\mu_F = \mu_R = M_Z.$$

One-loop weak correction: Numerical result

- Comparison with WZGRAD at 14 TeV

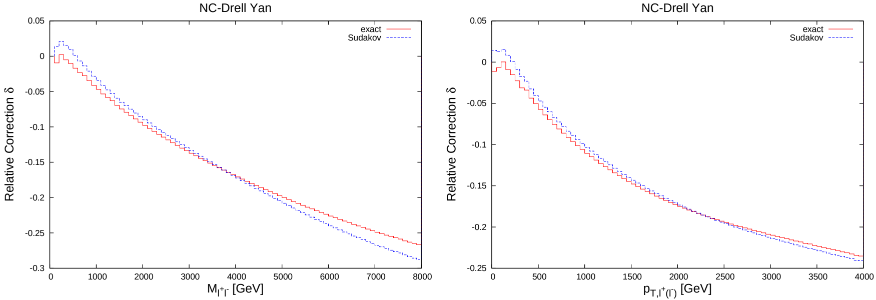
$$M_{l^+l^-} > 100\,\mathrm{GeV}, |p_{T,l^\pm}| > 20\,\mathrm{GeV}, |\eta_{l^\pm}| < 2.5$$

$$R = \frac{\sigma_{NLO} - \sigma_{LO}}{\sigma_{LO}}$$



Comparison: Sudakov approximation and exact calculation

- Invariant mass and transverse momentum distributions at LHC (14 TeV) with MCFM

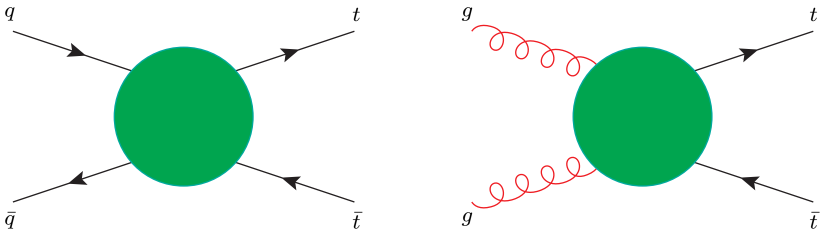


Summary to NC-DY

- A good exercise to start with
- To better understand and characterize the validity of the Sudakov approximation by comparing with the exact NLO calculation
- Have implemented both Sudakov approximation and exact NLO weak in MCFM
- Sudakov approximation shows good agreement with exact NLO weak

Sudakov approximation to $t\bar{t}$ production

Processes under consideration: $\bar{q}^\lambda_\rho q^\lambda_\rho t^\kappa \bar{t}^\kappa \to 0$ and $gg\, t^\kappa \bar{t}^\kappa \to 0$



- Chiralities to initial and final states
  - massless initial quarks(gluons) $\to$ chirality $=$ helicity, conserved during transportation,
  - massive final top quarks $\to$ chirality $\neq$ helicity, oscillating along the moving direction.
- Use projector to restore the weak corrections in the chiral coupling

Sudakov approximation to $t\bar{t}$ production

- Two ways to proceed the calculation
  1. break down the amplitude with chiralities ✓
  2. calculate the matrix element square directly ✓

Chiral Born

$$|\mathcal{M}|^2_{\mathrm{Born}} = |\mathcal{M}_{\mathrm{LL}}|^2 + |\mathcal{M}_{\mathrm{RR}}|^2 + |\mathcal{M}_{\mathrm{LR}}|^2 + |\mathcal{M}_{\mathrm{RL}}|^2,$$
$$|\mathcal{M}_{\mathrm{LL}}|^2 = |\mathcal{M}_{\mathrm{RR}}|^2, \quad |\mathcal{M}_{\mathrm{LR}}|^2 = |\mathcal{M}_{\mathrm{RL}}|^2$$

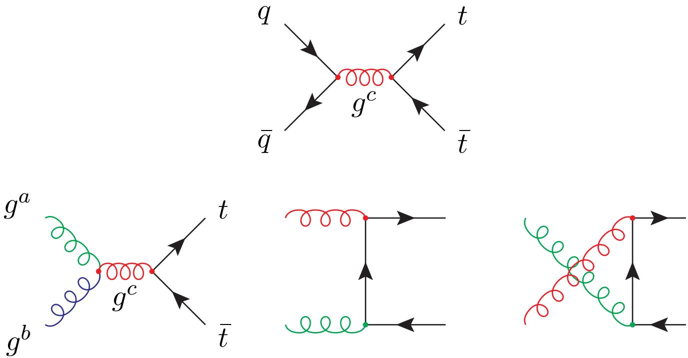
- Universal correction independent of chirality

$$\sum_{f^\sigma_\tau} \left[-C^{\mathrm{ew}}_{f^\sigma_\tau} (\mathrm{L}(\hat{s}) - 3 \cdot l_c) \right] |\mathcal{M}|^2_{\mathrm{Born}}$$

- Angular dependence and Yukawa enhanced terms
- No parameter renormalization

One-loop weak correction to $t\bar{t}$ production

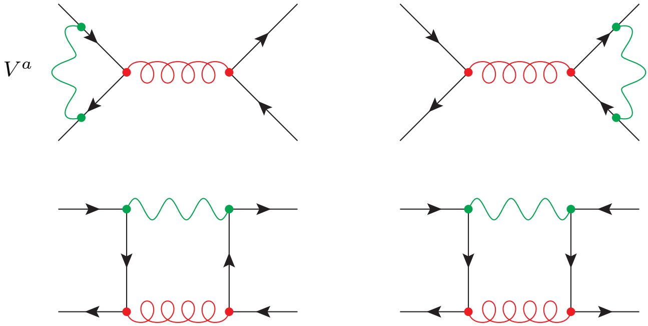
- Specific process at partonic level



Tree level strong production order of $\mathcal{O}(\alpha_s^2)$

One-loop weak correction to $t\bar{t}$ production

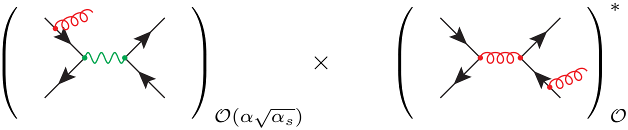
- Weak correction to quark-antiquark annhilation



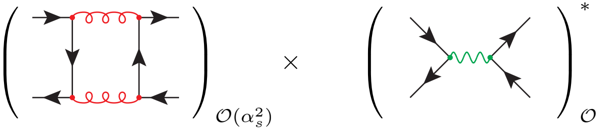
$$V^a = Z,\ W^{\pm}$$

One-loop weak correction to $t\bar{t}$ production

- Weak correction to quark-antiquark annhilation
  - difference between the strong mediated process and the pure weak process: IR divergence in virtual correction
  - real correction: gluon radiation from Z/g-mediated Born

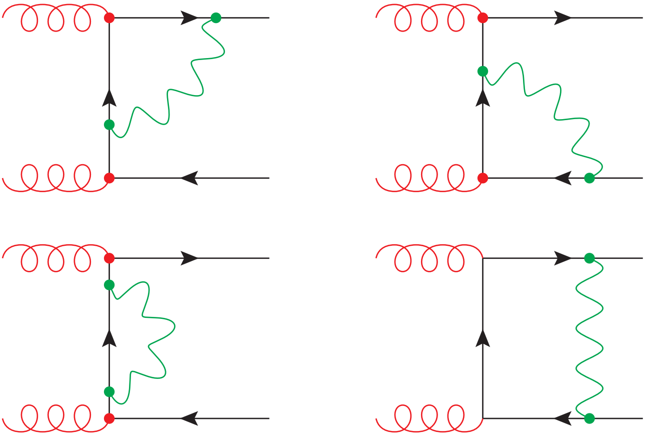


  - QCD box interferes with the Z-mediated Born (same order of $\mathcal{O}(\alpha\alpha_s^2)$)



One-loop weak correction to $t\bar{t}$ production

- Weak correction to gluon fusion



One-loop correction to $\mathrm{t\bar{t}}$ production: Numerical result

- Input parameters

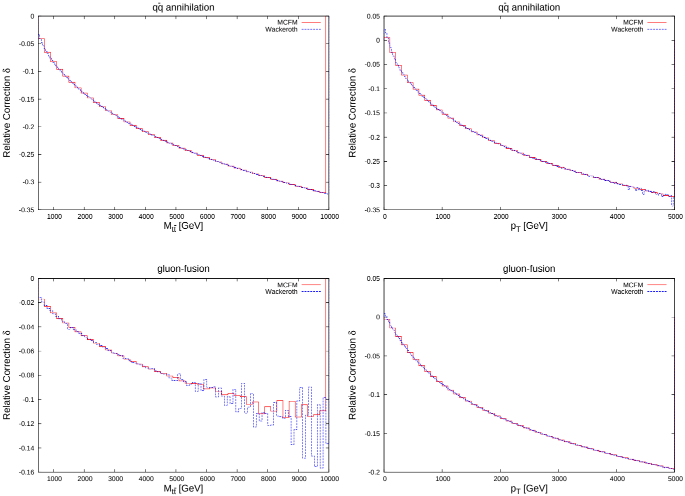
$$
\begin{aligned}
&M_Z = 91.1876\,\mathrm{GeV},\ M_W = 84.425\,\mathrm{GeV},\ M_H = 120\,\mathrm{GeV},\\
&m_b = 4.6\,\mathrm{GeV},\ m_t = 173.2\,\mathrm{GeV},\ \ s_W^2 = 0.2221236,\\
&\alpha = \alpha_\mu = 1/132.5605045,\ \alpha_s(2m_t) = 0.09897922,\\
&\mu_F = \mu_R = 2m_t.
\end{aligned}
$$

- The total cross sections

σ (fb)	$q\bar{q}$	gg	
$\mathcal{O}(\alpha_s^2)$	55408(9)	354251(66)	(MCFM)
LO	55386(18)	354254(47)	(Wackeroth)
$\mathcal{O}(\alpha\alpha_s^2)$	-1012.2(5)	-3887(1)	(MCFM)
NLO weak	-1011(1)	-3886(2)	(Wackeroth)

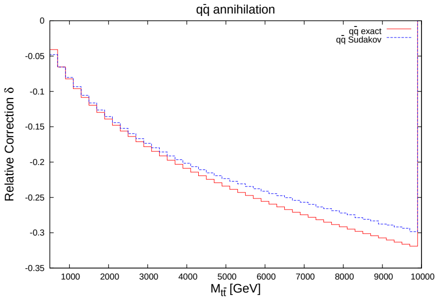
One-loop correction to $t\bar{t}$ production: Numerical result

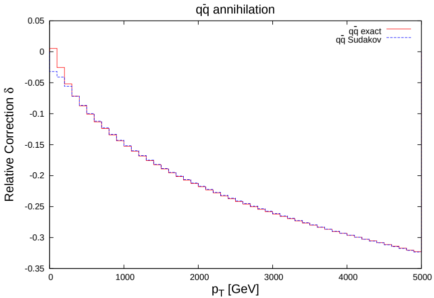
- Cross-check of the exact result at LHC = 14 TeV



Comparison with Sudakov approximation

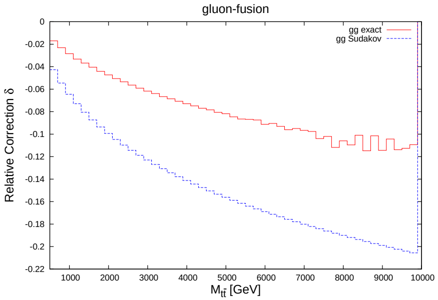
- Comparison between Sudakov approx and 1-loop exact calculation at LHC = 14 TeV with MCFM

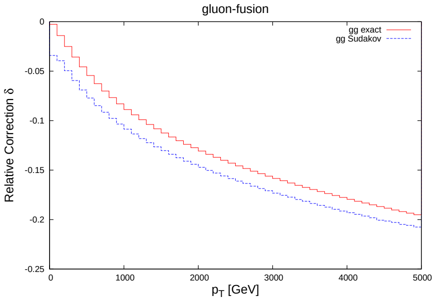




Comparison with Sudakov approximation

- Comparison between Sudakov approx and 1-loop exact calculation at LHC = 14 TeV with MCFM

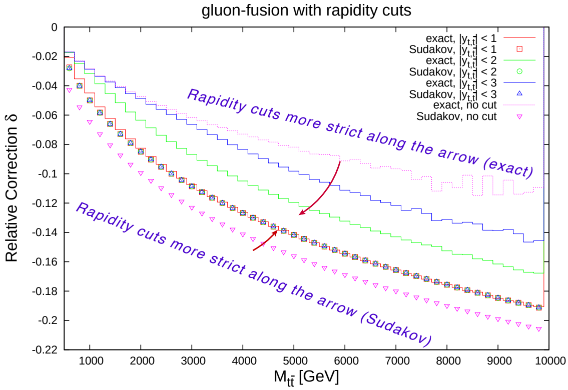




$$
\begin{aligned}
p_t &= (m_T \cosh y_t, p_T \sin\phi, p_T \cos\phi, m_T \sinh y_t), \\
p_{\bar{t}} &= (m_T \cosh y_{\bar{t}}, -p_T \sin\phi, -p_T \cos\phi, m_T \sinh y_{\bar{t}}), \\
M_{t\bar{t}}^2 &= 2m_t^2 + 2m_T^2 \cosh(y_t - y_{\bar{t}}) + 2p_T^2, \\
m_T &= \sqrt{p_T^2 + m_t^2}.
\end{aligned}
$$

Comparison with Sudakov approximation

- The invariant mass distributions with rapidity cuts; Sudakov approximation agrees well with the exact when $|y_{t,\bar{t}}| \lesssim 1$.



"..., it is clear that for the logarithmic approximation described be valid all Mandelstam variables $\hat{s}$, $\hat{t}$, $\hat{u}$ must be very large, condition which is obviously not fulfilled at small/large scattering angles." *[Weak corrections to gluon-induced top-antitop hadro-production]*

[S. Moretti *et al*, PLB639 (2006) 513]

"The gluon induced part, in contrast, is markedly angular dependent. For large $\hat{s}$ and small scattering angle the corrections are small, since the Sudakov-like behaviour cannot be expected in this case. At ninety degrees, in contrast, the Sudakov limit is applicable and the corrections become large." *[Weak Interactions in Top-Quark Pair Production at Hadron Colliders: An Update]*

J. H. Kühn *et al*, [arXiv:1305.5773]

Summary to $t\bar{t}$ production

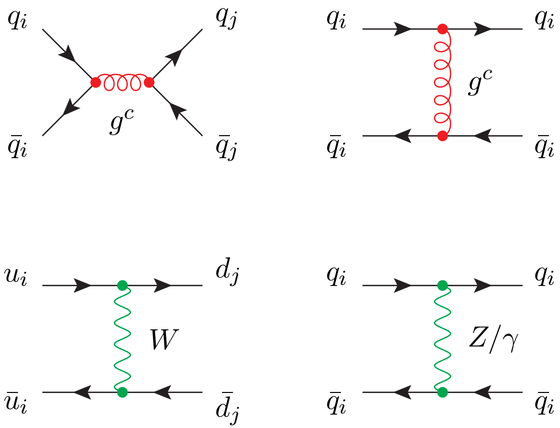
- We implement EW corrections to the top-pair production in MCFM, making the calculation accessible to the public.
- Both EW Sudakov approximation and exact weak NLO are implemented in MCFM.
- Sudakov approximation works much better in quark-antiquark annihilation channel, in contrary to gluon-fusion channel which has a obvious discrepancy between Sudakov approximation and exact NLO in invariant mass distribution due to the information of angular dependence is missing in Sudakov approximation.
- With a scattering angle cut to gluon-fusion channel, we are able to get an agreement between both calculations.

Dijet production

- Processes under consideration:
  - $\boxed{\text{quark-induced:}}$ $q_i\bar{q}_i \to q_j\bar{q}_j$, and its crossing symmetries such as $q_iq_j \to q_iq_j$, etc.
  - $\boxed{\text{gluon-induced:}}$ $gg \to q\bar{q}$, and its crossing symmetries such as $gq \to qg$, etc.
- Processes calculated directly:
  - $q_i\bar{q}_i \to q_j\bar{q}_j$, for both $i \neq j$ and $i = j$, respectively.
  - $gg \to q\bar{q}$
- The rest of the production processes is obtained via crossing symmetries of the directly calculated production

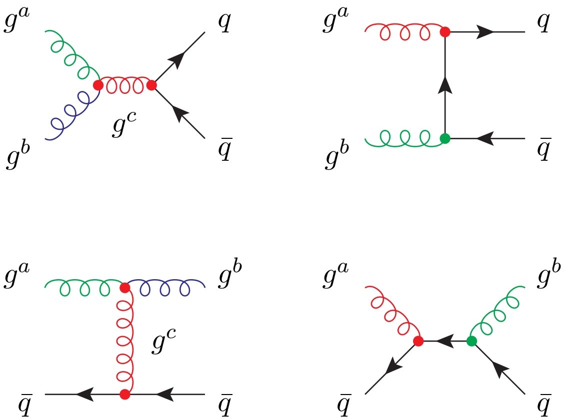
Dijet production

- Sample Born diagrams for the quark-induced production



Dijet production

- Sample Born diagrams for the gluon-induced production



Crossing symmetries

- $\boxed{\text{All quark-induced production via crossing symmetries}}$ $i \neq j$

1. $q_i\bar{q}_i \to q_j\bar{q}_j$, $\boxed{\text{direct calculation}}$
2. $q_iq_j \to q_iq_j$, $(2 \to 3,\ 3 \to 4,\ 4 \to 2;\ s \to t,\ t \to u,\ u \to s)$
3. $\bar{q}_iq_i \to q_j\bar{q}_j$, $(1 \leftrightarrow 2;\ t \leftrightarrow u)$
4. $\bar{q}_i\bar{q}_j \to \bar{q}_i\bar{q}_j$, $(1 \to 3,\ 3 \to 2,\ 2 \to 1;\ s \to t,\ t \to u,\ u \to s)$
5. $q_i\bar{q}_j \to q_i\bar{q}_j$, $(2 \leftrightarrow 3;\ s \leftrightarrow t)$
6. $\bar{q}_iq_j \to \bar{q}_iq_j$, $(1 \to 3,\ 3 \to 4,\ 4 \to 2,\ 2 \to 1;\ s \leftrightarrow t)$
7. $q_i\bar{q}_i \to q_i\bar{q}_i$, $\boxed{\text{direct calculation}}$
8. $\bar{q}_iq_i \to q_i\bar{q}_i$, $(1 \leftrightarrow 2;\ t \leftrightarrow u)$
9. $q_iq_i \to q_iq_i$, $(2 \to 3,\ 3 \to 4,\ 4 \to 2;\ s \to t,\ t \to u,\ u \to s)$
10. $\bar{q}_i\bar{q}_i \to \bar{q}_i\bar{q}_i$, $(1 \to 3,\ 3 \to 2,\ 2 \to 1;\ s \to t,\ t \to u,\ u \to s)$

where $12 \to 34$ denotes $q_i\bar{q}_i \to q_j\bar{q}_j$

Crossing symmetries

- All gluon-induced production via crossing symmetries

1. $gg \to q\bar{q}$, direct calculation
2. $gq \to qg$, $(2 \leftrightarrow 4;\ s \leftrightarrow u)$
3. $g\bar{q} \to \bar{q}g$, $(2 \to 4,\ 4 \to 3,\ 3 \to 2;\ s \to u,\ u \to t,\ t \to s)$
4. $qg \to qg$, $(1 \leftrightarrow 4;\ s \leftrightarrow t)$
5. $\bar{q}g \to \bar{q}g$, $(1 \to 2,\ 2 \to 4, 4 \to 3,\ 3 \to 1;\ s \leftrightarrow t)$
6. $q\bar{q} \to gg$, $(1 \leftrightarrow 3,\ 2 \leftrightarrow 4;\ t \leftrightarrow u)$
7. $\bar{q}q \to gg$ $(1 \leftrightarrow 4,\ 2 \leftrightarrow 3)$
8. $gg \to gg$, no weak correction

where $12 \to 34$ denotes $gg \to q\bar{q}$

work is in progress

Conclusion and outlook

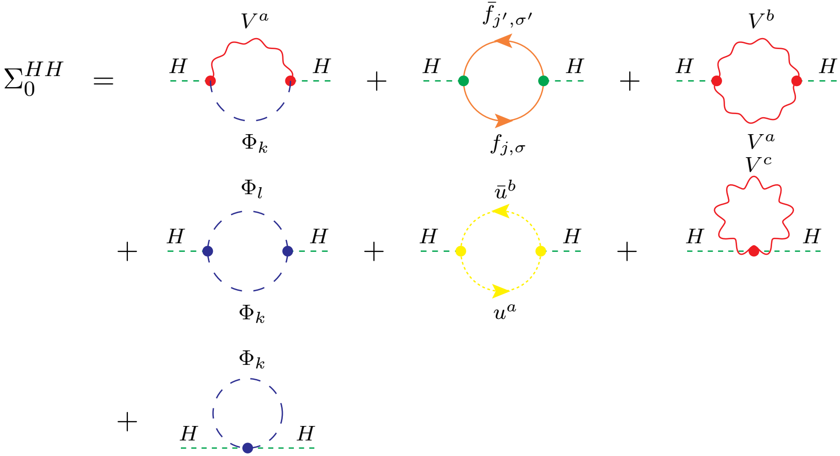
- The EW radiative corrections are very important at the LHC due to the Sudakov logarithmic terms.
- Implementation of the higher order EW corrections in MCFM makes these corrections available to the public.
- We have completed the implementation of both the Sudakov and exact weak NLO corrections to NC-DY and top-pair production into MCFM.
- The implementation of EW corrections to dijet production in MCFM is ongoing.
- We would like to continue, for instance, with implementation for ZZ production etc.

Backup formulism to Sudakov approximation

Backup Slides

Higgs external legs

- Self energy



Higgs external legs

- Counterterm

$$\delta\Sigma^{HH}(p^2) \quad = $$

$$= i\left(p^2\delta Z_{HH} - M_H^2\delta Z_{HH} - \delta M_H^2\right)$$

On Shell scheme renormalization s.t.

$$\hat{\Sigma}^{HH} = \Sigma_0^{HH} + \delta\Sigma^{HH},$$

$$\mathrm{Re}\frac{\partial\hat{\Sigma}^{HH}}{\partial p^2}\Big|_{p^2=M_H^2} = 1 \quad \Rightarrow \delta Z_{HH} = -\frac{\partial\Sigma_0^{HH}}{\partial p^2}\Big|_{p^2=M_H},$$

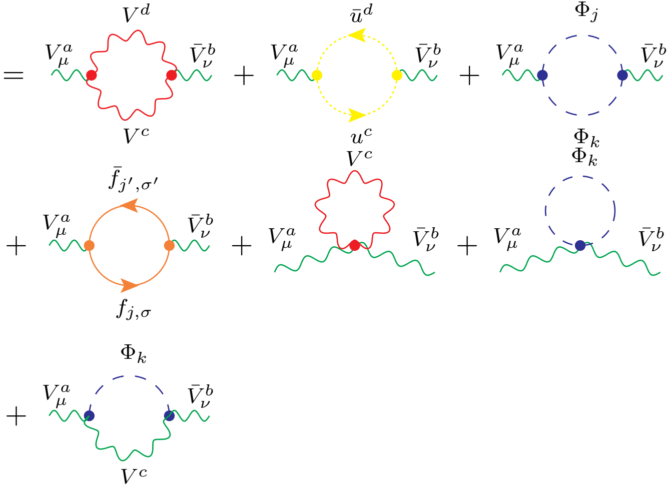
$$\mathrm{Re}\hat{\Sigma}^{HH}(p^2 = M_H^2) = 0 \quad \Rightarrow \delta M_H = \frac{\Sigma_0^{HH}(p^2 = M_H^2)}{1+\delta Z_{HH}},$$

$$\Rightarrow \delta Z_{HH} \overset{\mathrm{LA}}{=} \frac{\alpha}{4\pi}\left[2C_\Phi^{ew}\log\frac{\mu^2}{M_H^2} - \frac{N_c^t}{2s_W^2}\frac{m_t^2}{M_W^2}\log\frac{\mu^2}{M_{H,t}^2}\right]$$

Transverse gauge bosons

- Self energy

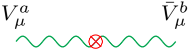
$$-i\Sigma^{V^a\bar{V}^b}_{\mu\nu,0} =$$



Transverse gauge bosons

- Counterterm

$$i\delta\Sigma_{\mu\nu}^{V^a\bar{V}^b} =$$



$$= -ig_{\mu\nu}(C_1 p^2 - C_2)$$

$V_\mu^a \bar{V}_\nu^b :$	C_1	C_2
$W^+W^- :$	δZ_W	$M_W^2 \delta Z_W + \delta M_W^2$
$ZZ :$	δZ_{ZZ}	$M_Z^2 \delta Z_{ZZ} + \delta M_Z^2$
$AZ :$	$\frac{1}{2}(\delta Z_{AZ} + \delta Z_{ZA})$	$M_Z^2 \frac{1}{2} \delta Z_{ZA}$
$AA :$	δZ_{AA}	0

On Shell scheme renormalization s.t.

$$\hat{\Sigma}_T^{V^a\bar{V}^b} = \Sigma_{T,0}^{V^a\bar{V}^b} + \delta\Sigma_T^{V^a\bar{V}^b}$$

$$\mathrm{Re}\frac{\partial \hat{\Sigma}_T^{V^a\bar{V}^b}(p^2)}{\partial p^2}\Big|_{p^2=M_{V^a}} = 0, \quad V^a = V^b$$

Transverse gauge bosons

On Shell scheme renormalization s.t.

$$\mathrm{Re}\hat{\Sigma}_T^W(M_W^2) = 0, \quad \mathrm{Re}\hat{\Sigma}_T^{ZZ}(M_Z^2) = 0, \quad \mathrm{Re}\hat{\Sigma}_T^{AZ}(M_z^2) = 0,$$
$$\mathrm{Re}\hat{\Sigma}_T^{AZ}(0) = 0, \quad \mathrm{Re}\hat{\Sigma}_T^{AA}(0) = 0,$$

$$\Rightarrow \delta M_{V^a}^2 = \mathrm{Re}\Sigma_{T,0}^{V^a\bar{V}^a}(M_{V^a}^2), \quad \delta Z_{V^aV^a} = -\mathrm{Re}\frac{\partial \Sigma_{T,0}^{V^a\bar{V}^a}(p^2)}{\partial p^2}\Big|_{p^2=M_{V^a}^2},$$

$$\delta Z_{V^aV^b} = \frac{2\mathrm{Re}\Sigma_{T,0}^{V^a\bar{V}^b}(M_{V^b}^2)}{M_{V^a}^2 - M_{V^b}^2}, \quad V^a \neq V^b.$$

$$\delta Z_{V^aV^b} \overset{\mathrm{LA}}{=} \frac{\alpha}{4\pi}\Big\{ \left[b_{V^aV^b}^{\mathrm{ew}} - 2C_{V^aV^b}^{\mathrm{ew}} + b_{AZ}^{\mathrm{ew}} E_{V^aV^b}\right] \log\frac{\mu^2}{M_W^2}$$
$$+2\delta_{V^aV^b} Q_{V^a}^2 \log\frac{M_W^2}{\lambda^2}\Big\} - \delta_{V^aA}\delta_{V^bA}\Delta\alpha(M_W^2) + \delta Z_{V^aV^b}^{H,t},$$

Transverse gauge bosons

On Shell scheme renormalization s.t.

$$\delta Z^{H,t}_{V^aV^b} = \frac{\alpha}{4\pi}\left\{\delta_{V^aV^b}\frac{M^2_{V^a}}{12s^2_{\mathrm{W}}M^2_W}\log\frac{M^2_H}{M^2_W} + \frac{2N^t_{\mathrm{C}}}{3}T_{V^aV^b}\log\frac{M^2_t}{M^2_W}\right\},$$

$$\Delta\alpha(M^2_W) = \frac{\alpha}{3\pi}\sum_{f_{j,\sigma}\neq t} N^f_{\mathrm{C}}Q^2_{f_{j,\sigma}}\log\frac{M^2_W}{m^2_{f_{j,\sigma}}}$$

$$T_{V^aV^b} = \sum_{\kappa=\mathrm{R,L}}\left[\left(I^{V^a}I^{\bar{V}^b} + Q^2_{V^a}I^{\bar{V}^b}I^{V^a}\right)_{\mathrm{t}^\kappa\mathrm{t}^\kappa} + \left(I^AI^Z\right)_{\mathrm{t}^\kappa\mathrm{t}^\kappa}E_{V^aV^b}\right]$$

$$E_{AZ} = -E_{ZA} = 1,$$

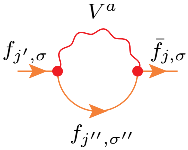
$$b^{\mathrm{ew}}_{V^aV^b} := \frac{11}{3}D^{\mathrm{ew}}_{V^aV^b}(V) - \frac{1}{6}D^{\mathrm{ew}}_{V^aV^b}(\Phi) - \frac{2}{3}\sum_{f=Q,L}\sum_{j=1,2,3}N^f_{\mathrm{C}}\sum_{\lambda=\mathrm{R,L}}D^{\mathrm{ew}}_{V^aV^b}(f^\lambda_j),$$

$$D^{\mathrm{ew}}_{V^aV^b}(\varphi) := \mathrm{Tr}_\varphi\left\{I^{\bar{V}^a}I^{V^b}\right\} = \sum_{\varphi_i,\varphi_{i'}} I^{\bar{V}^a}_{\varphi_i\varphi_{i'}}I^{V^b}_{\varphi_{i'}\varphi_i}.$$

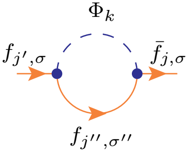
Chiral fermions

- Self energy

$$i\Sigma_0^{\Psi_{j',\sigma}\bar{\Psi}_{j,\sigma}} =$$



$+$



- Counterterm

$$i\delta\Sigma^{f_{j'}f_j}(p^2) =$$

$$= i\left[(C_L\cdot\omega_- + C_R\cdot\omega_+)\not{p} - (C_S^-\cdot\omega_- + C_S^+\cdot\omega_+)\right]$$

$$C_L = \frac{1}{2}\left(\delta Z_{jj'}^{f^L} + \delta Z_{jj'}^{f^L\dagger}\right), \qquad C_R = \frac{1}{2}\left(\delta Z_{jj'}^{f^R} + \delta Z_{jj'}^{f^R\dagger}\right),$$

$$C_S^- = m_{f_j}\frac{1}{2}\delta Z_{jj'}^{f^L} + m_{f_{j'}}\frac{1}{2}\delta Z_{jj'}^{f^R\dagger} + \delta_{jj'}\delta m_{f_j},$$

$$C_S^+ = m_{f_j}\frac{1}{2}\delta Z_{jj'}^{f^R} + m_{f_{j'}}\frac{1}{2}\delta Z_{jj'}^{f^L\dagger} + \delta_{jj'}\delta m_{f_j}.$$

Chiral fermions

On Shell scheme renormalization s.t.

$$\hat{\Sigma}^{f^\sigma}_{jj'} = \Sigma^{f^\sigma}_{jj',0} + \delta\Sigma^{f^\sigma}_{jj'},$$

$$m_{f_{j'}} \mathrm{Re}\hat{\Sigma}^{f^{L,R}}_{jj'}(m^2_{f_{j'}}) + m_{f_j} \mathrm{Re}\hat{\Sigma}^{f^S}_{jj'}(m^2_{f_{j'}}) = 0,$$

$$\mathrm{Re}\hat{\Sigma}^{f^L}_{jj}(m^2_{f_j}) + \mathrm{Re}\hat{\Sigma}^{f^R}_{jj}(m^2_{f_j}) +$$

$$2m^2_{f_j} \frac{\partial}{\partial p^2} \left(\mathrm{Re}\hat{\Sigma}^{f^L}_{jj}(p^2) + \mathrm{Re}\hat{\Sigma}^{f^R}_{jj}(p^2) + 2\mathrm{Re}\hat{\Sigma}^{f^S}_{jj}(p^2) \right) \Big|_{p^2 = m^2_{f_j}} = 0,$$

$$\Rightarrow \delta m_{f_j} = \frac{1}{2} \mathrm{Re} \left(\Sigma^{f^L}_{jj}(m^2_{f_j}) + \Sigma^{f^R}_{jj}(m^2_{f_j}) + 2\Sigma^{f^S}_{jj}(m^2_{f_j}) \right),$$

$$\delta Z^{f^{L,R}}_{jj} = -\mathrm{Re}\Sigma^{f^{L,R}}_{jj}(m^2_{f_j})$$

$$-m^2_{f_j} \frac{\partial}{\partial p^2} \mathrm{Re} \left[\Sigma^{f^{L,L}}_{jj}(p^2) + \Sigma^{f^{R,R}}_{jj}(p^2) + 2\Sigma^{f^{S,S}}_{jj}(p^2) \right] \Big|_{p^2 = m^2_{f_j}},$$

Chiral fermions

On Shell scheme renormalization s.t.

$$\delta Z^{f^{L,R}}_{jj'} = \frac{2}{m^2_{f_j} - m^2_{f_{j'}}}$$

$$\times\mathrm{Re}\left[m_{f^2_{j'}}\Sigma^{f^{L,R}}_{jj'}(m^2_{f^{j'}})+m_{f_j}m_{f_{j'}}\Sigma^{f^{R,L}}_{jj'}(m^2_{f_{j'}})+(m^2_{f_j}+m^2_{f_{j'}})\Sigma^{f^{S,S}}_{jj'}(m^2_{f_{j'}})\right],$$

$$\Rightarrow \delta Z_{f^\kappa_{j,\sigma}f^\kappa_{j',\sigma}} \sim \Sigma_0^{f_{j',\sigma}\bar{f}_{j,\sigma}}(m^2_{f^\kappa_{j,\sigma}}) \propto \underline{\underline{\delta_{fQ}\delta_{\sigma-}\mathbf{V}^\dagger_{j3}\mathbf{V}_{3j'}\log\frac{M^2_W}{m^2_t}}}, \quad (j \neq j')$$

non-vanishing only if involving virtual top, but the largest order is

$$\frac{\alpha}{4\pi}\frac{m^2_t}{s^2_W M^2_W}\left|\mathbf{V}^\dagger_{j3}\mathbf{V}_{3j'}\right|\log\frac{m^2_t}{M^2_W} < 10^{-3}, \Rightarrow \boxed{\delta Z_{f^\kappa_{j,\sigma}f^\kappa_{j',\sigma}} \overset{\mathrm{LA}}{=} 0,} \text{ for } j \neq j'$$

$$\Rightarrow \delta Z_{f^\kappa_{j,\sigma}f^\kappa_{j,\sigma}} \overset{\mathrm{LA}}{=} \frac{\alpha}{4\pi}\left[-C^\kappa_{f_{j,\sigma}}\log\frac{\mu^2}{M^2_W} + Q_{f^2_{j,\sigma}}\left(2\log\frac{M^2_W}{\lambda^2} - 3\log\frac{M^2_W}{m^2_{f_{j,\sigma}}}\right)\right] + \delta Z^{top}_{f^\kappa_{j,\sigma}},$$

Longitudinally gauge bosons

Goldstone-boson equivalence theorem (GBET)

$$\mathcal{M}^{V_{\mathrm{L}}^{a_1}\ldots V_{\mathrm{L}}^{a_m}\varphi_{i_1}\cdots\varphi_{i_n}}(q_1,\ldots,q_m,p_1,\ldots,p_n) =$$
$$= \left[\prod_{k=1}^{m} i^{(1-Q_{V^{a_k}})} A^{V^{a_k}}\right] \mathcal{M}^{\Phi_{a_1}\ldots\Phi_{a_m}\varphi_{i_1}\cdots\varphi_{i_n}}(q_1,\ldots,q_m,p_1,\ldots,p_n)$$
$$+\mathcal{O}(ME^{d-1}), \quad A^{V^a} = 1 + \delta A^{V^a}$$

d: mass dim of the matrix element, $E \sim \sqrt{s}$

One-loop correction to GBET LO

1. correction to GBET itself

$$\delta A^{V^a} = -\frac{\Sigma_{\mathrm{L},0}^{V^a\bar{V}^a}(p^2)}{M^2_{V^a,0}} + i^{(1+Q_{V^a})}\frac{\Sigma_{\mathrm{L},0}^{V^a\Phi^+}(p^2)}{M^2_{V^a}} + \frac{1}{2}\frac{\delta M^2_{V^a}}{M^2_{V^a}} + \frac{1}{2}\delta Z_{V^aV^a}$$

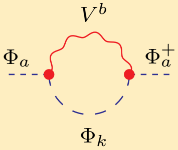
Longitudinally gauge bosons

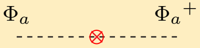
One-loop correction to GBET LO

1. correction to GBET itself

$$\delta A^{V^a} \overset{\text{LA}}{=} \frac{\alpha}{4\pi}\Bigg\{C_\Phi^{\text{ew}} \log\frac{\mu^2}{M_W^2} - \frac{N_\text{C}^t}{s_\text{W}^2}\frac{m_t^2}{M_W^2}\log\frac{\mu^2}{m_t^2} + Q_{V^a}^2 \log\frac{M_W^2}{\lambda^2} + \frac{3M_{V^a}^2}{8s_\text{W}^2 M_W^2}\log\frac{M_H^2}{M_W^2},$$

2. FRC ($\frac{1}{2}\delta Z_{\Phi_a\Phi_a}$) to GBET external scalar bosons

$i\Sigma_0^{\Phi_a{\Phi_a}^+} =$ 

$+ \quad \ldots,\qquad \text{CT} =$ 

$$\delta Z_{\Phi_a\Phi_a} \overset{\text{LA}}{=} \frac{\alpha}{2\pi}\left\{C_\Phi^{\text{ew}} \log\frac{\mu^2}{M_W^2} - \frac{N_\text{C}^t}{4s_\text{W}^2}\frac{m_t^2}{M_W^2}\log\frac{\mu^2}{m_t^2} + \frac{M_{V^a}^2}{4s_\text{W}^2 M_W^2}\log\frac{M_H^2}{M_W^2}\right\}.$$